THE LONGITUDINAL LAMINAR FLOW OF A LIQUID IN A BUNDLE OF RODS

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Analog and digital methods have been applied to the problem of the longitudinal flow of a viscous incompressible liquid through bundles of rods in square or triangular array.

The active zone in nuclear reactors is frequently made up of cylindrical heat-evolving elements which are streamlined by a longitudinal flow of a heat carrier. The calculation of the characteristics for the laminar flow of a liquid in such a system is of practical interest; however, the literature contains very few references devoted to this problem.

Here we will present the results obtained in studies of the laminar flow of a liquid in the space between the heat evolving rods positioned at the corners of an equilateral triangle or square. The problem of determining the velocity fields in the liquid has been resolved both by analog and digital methods.

The equation describing the laminar motion of the liquid has the form

$$\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=\frac{\partial p}{\partial z} \tag{1}$$

with the boundary condition

$$u(\Gamma)=0, \tag{2}$$

where Γ is the liquid-solid interface.

When the right-hand member of Eq. (1) is constant, it can be simulated with ac current through the use of a flat capacitor [1].

The investigation was carried out on the model of an elementary symmetry cell ABCDA (Fig. 1). In measuring the potentials on the model, we divided the entire investigated region into a rather large number of elementary units. Depending on the relative pitch h, the side of such an elementary unit was 0.01–0.02 of the rod radius. The value of the potential was determined at the center of each unit. The mean value was determined in a manner as to account for the specific weight of incomplete boundary unit cells. The number of elementary units within the symmetry cell ABCDA amounted to several hundred.

A finite-difference method was used for the numerical solution of Eq. (1). The calculation was carried out in cylindrical coordinates. The corner spacing ranged from 1°40' for dense packing to 5° for a system with a pitch of 1.5, and along the radius it ranged from $\Delta r = 0.025R$ for h = 1.0 to $\Delta r = 0.125R$ for h = 1.5.

The results of the solution and comparison with other data are shown in Figs. 2–4.

Figure 2 shows the distribution of velocities over the symmetry line AF (or, what is the same, along the line ABC in Fig. 1) for rods in triangular array. The agreement between the results of analog and digital calculation is excellent, the divergence not exceeding several percent. Moreover, Fig. 2 shows the result of the numerical solution for h = 1.0 [10], which is also in good agreement with the authors' data.

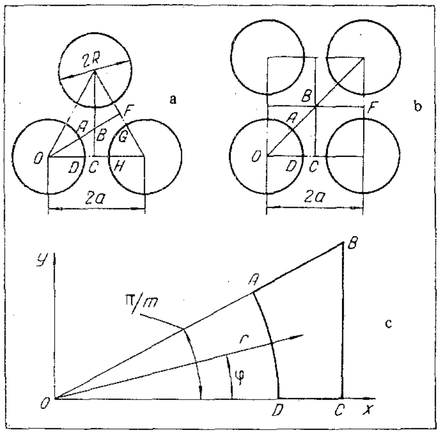

Fig. 1. Arrangement of rods in periodic array; a) triangular arrangement; b) square arrangement: c) typical element.

One of the earliest theoretical solutions for the problem under consideration in the case of rods in square array was offered by Emersleben [2]. This is an approximate solution, and boundary condition (2) is satisfied only with some error. The Emersleben solution is exact for flow in the space between rods whose lateral cross section over a great portion of the flow is somewhat compressed in the direction of the rod centers. This effect is all the more pronounced the smaller the relative spacing of the system.

For a dense packing, the maximum velocity found from the analytical solution [2] is greater by a factor of approximately two than the corresponding velocity obtained by analog procedures. This divergence is a consequence of the assumptions made in the solution of the problem. For a spacing h = 1.5, the analytical solution differs insignificantly from that found by the analog method.

For circular channels, the calculation of the resistance factor proceeds with the use of the formula $\lambda = 64/\mathrm{Re}$, so that we can present the friction coefficient for the case of laminar flow in rod bundles in the form

$$\lambda = A\frac{64}{\mathrm{Re}}. \tag{3}$$

The coefficient A as a function of spacing [pitch] and the type of array is shown in Fig. 3. In processing these results, we chose the hydraulic diameter of the

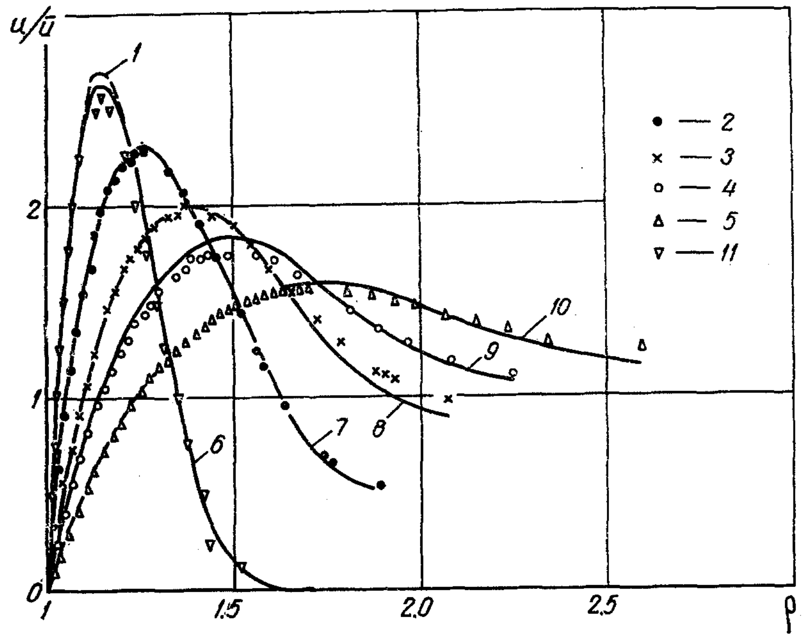

Fig. 2. Velocity distribution over the symmetry line ABF for triangular arrangement of rods· 1-5) electric simulation for h = 1.0; 1.1; 1.2; 1.3; 1.5; 6-10) numerical solution for h = 1.0; 1.1; 1.2; 1.3; 1.5; 11) numerical solution [10] for h = 1.0.

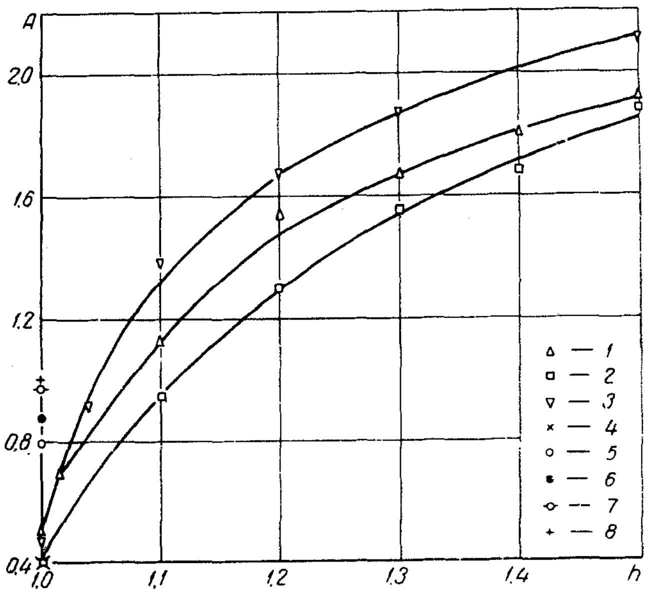

Fig. 3. Factor A versus pitch: 1, 2) electric simulation for triangular and square lattices; 3) numerical value, triangular lattice; 4) experiment [4]; 5, 6, 7) experiment [5], Re = 1270, 1610, 2000, triangular lattice; 8) experiment [5], Re = 2300, square lattice.

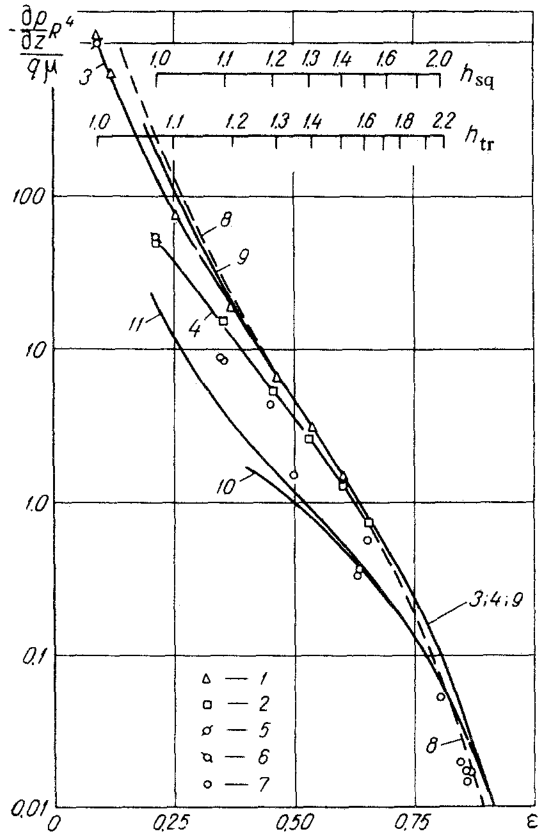

Fig. 4. Hydraulic resistance versus porosity: 1, 2) electric simulation, triangular, and square arrangement of rods (m = 6, m = 4); 3, 4) analytical solution (m = 6; m = 4); 5, 6) experiments [4] (m = 6, m = 4); 7) experiments [4], chaotic arrangement of rods; 8) according to "Heat Transfer Manuals" [9] for m = 6; 9) annular flow, calculation according to formula (8); 10) solution [2], calculation with respect to true porosity of system of deformed rods; 11) solution [2], calculation with respect to fictitious porosity of rod system with radius R = OD (Fig. 1).

system as the decisive dimension. The resistance factor for the triangular array—found by numerical solution of the equation (curve 3 in Fig. 3)—lies higher than the data derived from the analog method because of the error in the numerical method. However, in the case of dense packing we find that the resistance factors determined by analog and numerical methods are in agreement.

Let us compare these results with those obtained experimentally. Reference [4] cites the results from an experimental determination of the resistance factor in the case of a laminar flow of air parallel to cylindrical surfaces. For densely packed rods in square array, according to [4] we have A = 0.414, which differs little from the result (A = 0.40) given by the analog method. The authors of [11]—solving the problem numerically—for these same conditions obtained a value of A = 0.406.

For a triangular rod array the author of [4] derived values of A = 0.403 and A = 0.410, which is approximately 20% lower than the values achieved by the analog and numerical methods. The authors know of other experiments carried out with water [5]. Figure 3 shows the results of these experiments for a triangular rod array. The data of [5] are higher by a factor of 2–2.5 than the corresponding results of the numerical calculation, the analog calculation, and the experiments with air [4], and this divergence increases as the Re number grows. It is possible that in the experiments described in [5] the flow regime was not laminar, and this could lead to an exaggeration of the resistance factor. The Reynolds number calculated from the hydraulic diameter ranges in [5] from 1270 to 1300, while in the experiments of [4] it varies within limits of 10 to 100. It can therefore be assumed that the flow in reference [4] was stable and laminar.

For practical calculations and to compare the results from triangular and square rod arrays, it is more convenient to give the hydraulic resistance data in the form of a dimensionless pressure difference, referred to a unit of liquid flow rate. We have taken the porosity ε of the system as the independent variable, this quantity representing the ratio of the cross-sectional area to the over-all area occupied by the liquid and the rods. For convenience in comparing the triangular and square rod arrays, we have referred the pressure difference to the flow rate for a single rod. Curves 3 and 4 in Fig. 4 show the results of an approximate analytical solution [3], which has been derived by a method of discrete satisfaction of the boundary conditions, the application of this method having been treated in [6, 7]. We see that the analog solution and the analytical solution of [3] agree. For a porosity above $\varepsilon = 0.5$ the pressure difference for the triangular and square rod arrays becomes identical. The Emersleben solution (curves 10 and 11) yields lowered results.*

The experimental data of Sullivan [4] agree with the analog method only for densely packed square and triangular arrays. For higher porosity values, the experiments are smaller than the analog calculations by factors of one-and-a-half to two. One of the factors responsible for this divergence is the random positioning of the rods in the Sullivan experiment. With this array, local variations in the porosity of the material are unavoidable, and the reduction in the liquid flow rate in the restricted regions is more than offset by the increase in the flow rate through the region exhibiting greater porosity.

The authors of the Handbook of Heat Transfer (9) recommend the use of the following relationship to calculate the hydraulic resistance for the laminar flow in the intertube space:

$$\lambda \,\mathrm{Re} = 121. \tag{4}$$

We see from Fig. 4 that formula (4) yields exaggerated values for the pressure difference when the pitch is small, and exaggerated values when the pitch is large. The calculation shows that the result is exaggerated by a factor of almost 5 for a dense packing. With a pitch range from h = 1.2 to h = 1.8 relationship (4) yields an error of ±20%.

The analytical solution for the one-dimensional problem of longitudinal streamlining of the rods can be used as an approximation formula to calculate the resistance factor and the pressure difference. It is assumed in this solution that the velocity is a function exclusively of the radius and is independent of the angle. It is also assumed that the symmetry line at which the velocity gradient vanishes passes along a circle at a distance a from the rod center, so that the area of the ring between the rod and this circle is equal in magnitude to the cross-sectional area referred to a single rod in a real system. The analytical solution yields the following expression for the pressure difference:

$$\frac{(-\partial p/\partial z)\,R^4}{\mu q} = \frac{4(1-\varepsilon)^2}{\pi\left(\ln\dfrac{1}{1-\varepsilon} - \varepsilon - \varepsilon^2/2\right)}, \tag{5}$$

where

$$\varepsilon = \frac{\pi a^2 - \pi R^2}{\pi a^2} = 1 - \frac{R^2}{a^2}. \tag{6}$$

The relationship between the porosity and the pitch is given by

$$\varepsilon = 1 - \frac{\pi}{mh^2}\,\mathrm{tg}\,\frac{\pi}{m}. \tag{7}$$

To analyze the nature of the flow and to evaluate the interaction between the rods, we have determined the distribution of the tangential stresses τ over the circumference of the rod. Let the distribution of the potential on the model along the arc $r = a_m$ be $v(a_m, \varphi)$. The distribution of the tangential stresses over the circumference of the rod can then be derived from the analytical solution of Eq. (1) by the method of separation of variables:

$$\tau(R, \varphi)/\bar{\tau}(R) =$$

*Data taken from [8].

$$= 1 - \frac{2m^2\left(1 - h^2 + \frac{2mh^2}{\pi}\,\mathrm{tg}\,\frac{\pi}{m}\,\ln h\right)}{\pi\left(\frac{h^2 m}{\pi}\,\mathrm{tg}\,\frac{\pi}{m} - 1\right)\bar{v}(a_m)} \times$$

$$\times \sum_{k=1}^{\infty} \frac{kh^{-mk}\cos mk\varphi}{1 - h^{-2mk}} \int_0^{\pi/m} v(a_m\varphi)\cos mk\varphi\, d\varphi. \qquad (8)$$

The results of the simulation, reduced in accordance with formula (8), are in good agreement with the analytical solution [3].

The calculation shows that if the rods are positioned close to each other they exert significant influence on each other. A change in the tangential stresses in terms of the angle results in the case of a square array in pressure differences that are greater than in the case of a triangular array.

NOTATION

p is the pressure; q is the flow rate per rod; Re is the Reynolds number; u is the local velocity; u is the mean velocity; x and y are the transverse coordinates; z is the longitudinal coordinate; μ is the dynamic viscosity; ρ is the dimensionless instantaneous radius. Subscripts: sq is the square arrangement of rods; m is the model; tr is the triangular arrangement of rods.

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